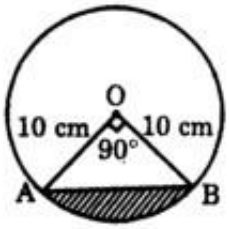
Areas Related To Circles

102) A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding minor segment?

2014/2015 [3 Marks]

See figure.



Area of the minor segment AB

$\Rightarrow$ Area of sector OAB – Area of $\triangle$OAB

$\Rightarrow \frac{90^\circ}{360^\circ} \times \pi r^2$ – Area of ΔOAB

$\Rightarrow [\frac{1}{4} \times \frac{22}{7} \times 10 \times 10 - \frac{1}{2} \times 10 \times 10]\ cm^2$

$\Rightarrow [\frac{25 \times 22}{7} - 50]\ cm^2 = [\frac{550 - 350}{7}] cm^2$

$\Rightarrow \frac{200}{7}\ cm^2 = 28\frac{4}{7}\ cm^2.$

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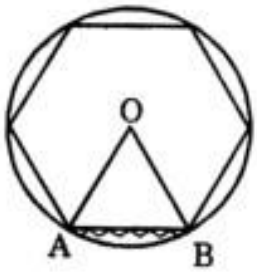
103) A round table cover has six equal designs as shown in the given figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs. 0.35 per cm^2?

2011/2015 [4 Marks]



Designs are equal ⇒ chords are equal and equal chords subtend equal angles at the centre.

$\Rightarrow$ Angles at the centre $= \frac{360^\circ}{6} = 60^\circ$

$\Rightarrow$ All the angles of ΔOAB are of 60°.
(Angles opposite to the equal sides are equal.)

$\Rightarrow$ ΔOAB is an equilateral triangle.
Area of one design
$= \text{ar(Sector AOB)} - \text{ar}(\Delta AOB)$

$= [\frac{60}{360} \times \frac{22}{7} \times 28 \times 28 - \frac{1}{2} \times 28 \times 28 \times \frac{\sqrt{3}}{2}]$

$= [\frac{1232}{3} - 339.5]\ cm^2 = 71.17\ cm^2$

$\therefore$ Area of 6 such designs $= 71.17 \times 6 = 427.02\ cm^2$
Cost of making such designs at the rate of Rs. 0.35 cm^2.

$= \text{Rs.}\ \frac{427.02 \times 35}{100} = Rs.\ 149.46$

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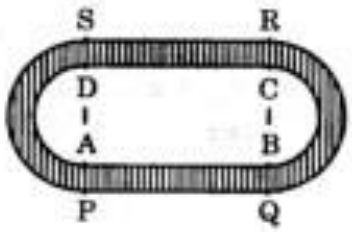
104) A 21 m wide athletic track consists of two straight sections 150 m long joining semi circular ends whose diameters are 84 m each (see figure). Find the area of the track? (Use $\pi = \frac{22}{7}$ and $\sqrt{3} = 1.73$)



2011/2015 [4 Marks]

Area of the track

$\Rightarrow$ =
Area of rectangle PQRS – Area of rectangle ABCD +
[Area of semicircle of radius 42 cm – Area of semicircle of diameter (42 – 21) m] × 2

$\Rightarrow$ $= \{150 \times 84 - 150 \times (84 - 21 - 21)\}m^2 + 2 \times \{\frac{1}{2} \times \frac{22}{7} \times (42)^2 - \frac{1}{2} \times \frac{22}{7} \times (21)^2\}\ m^2$

$\Rightarrow$ $= 150 \times (84 - 42)\ m^2 + 2 \times \frac{1}{2}[\frac{22}{7}(63)(21)]\ m^2$

$\Rightarrow$ $= 150 \times 42\ m^2 + (22 \times 9 \times 21)\ m^2$

$\Rightarrow$ $= 42(150 + 99)\ m^2 = 42 \times 249\ m^2$

$\Rightarrow$ $= 10458\ m^2$.

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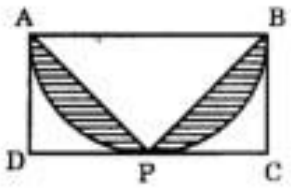
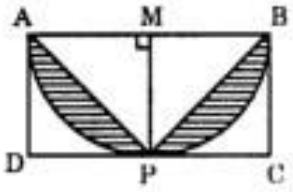
105) In the given figure, ABCD is a rectangle of length $10\sqrt{2}$ cm and breadth $5\sqrt{2}$ cm. If APB is an isosceles triangle inscribed in the semicircle with diameter AB, find the area of the shaded region?



2011/2015 [4 Marks]



Draw PM $\perp$ AB
As AP = BP, so M is the mid – point of AB
SO,

$\Rightarrow$ PM = $5\sqrt{2}$ cm

Therefore,

$\Rightarrow$ ar(ΔAPB) = $\frac{1}{2}$AB $\times$ PM = $\frac{1}{2}10\sqrt{2} \times 5\sqrt{2}$ cm^2

$\Rightarrow$ = 50 cm^2

$\Rightarrow$ Area of semicircle APB = $\frac{1}{2} \times \frac{22}{7} \times (5\sqrt{2})$ cm^2

$\Rightarrow$ = $\frac{1}{2} \times \frac{22}{7} \times 25 \times 2$ cm^2

$\Rightarrow$ = $\frac{550}{7}$ cm^2

So, area of the shaded region =Area of semicircle $-$ Area of triangle

$\Rightarrow$ = $[\frac{550}{7} - 50]$ cm^2 = $\frac{200}{7}$ cm^2
